2024.M29

2024L003A1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 1

Higher Level

Friday 7 June Afternoon 2:00 - 4:30

300 marks

Examination Number

Date of Birth / /

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) Solve the following equation for $n \in \mathbb{N}$:

$$n - 3 = \sqrt{3n + 1}$$

(b) Write the following expression as a single fraction in terms of t :

$$\frac{4}{2t + 1} - \frac{7}{12t}$$

(c) Solve the following simultaneous equations for $x, y, w \in \mathbb{Z}$:

$$\begin{aligned} x + 2y &= 143 \\ y + 3w &= -74 \\ 4x + 5w &= 4 \end{aligned}$$

Question 2 **(30 marks)**

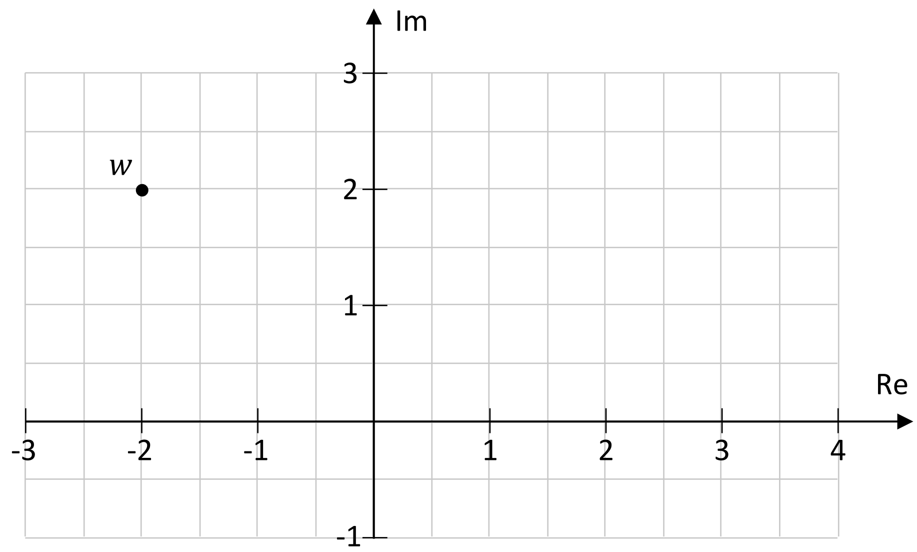
In this question, $i^2 = -1$.

(a) Find the two solutions of the following equation in z, where z is a complex number.
Give each answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

$$z^2 + 12z + 261 = 0$$

(b) Use **de Moivre's theorem** to write $\left(1 - \sqrt{3}\, i\right)^9$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

(c) The point $w = -2 + 2i$ is shown in the Argand diagram below.



(i) **Plot** and **label** the complex number $u = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ on the diagram, as accurately as possible.

(ii) The complex number o is $0 + 0i$.

Find the size of the angle $\angle wou$. Give your answer in **radians**.

Question 3 **(30 marks)**

(a) Find the integral:

$$\int \cos 6x \; dx$$

(b) The function f is defined for $x \in \mathbb{R}$ as:

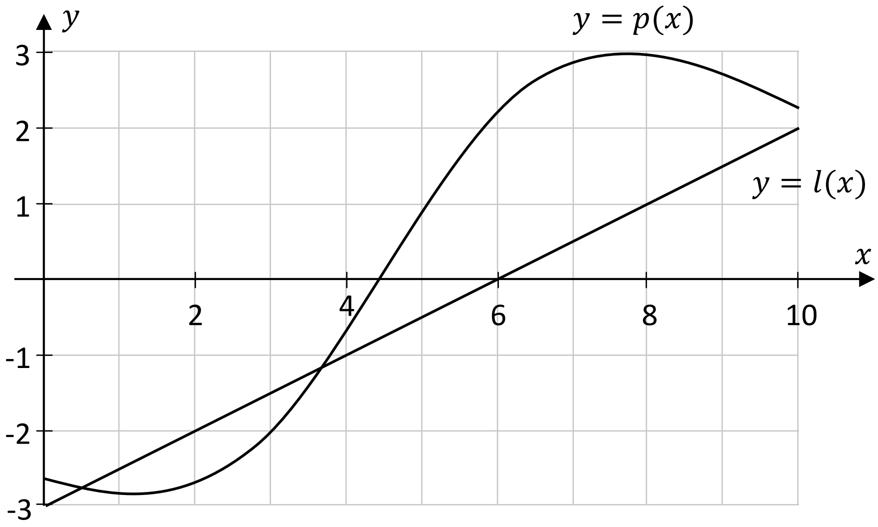
$$f(x) = 2x^3 - 9x^2 + 5x - 11$$

(i) Find the equation of the tangent to the graph of f at the point where $x = 2$.
You do **not** need to simplify your answer.

(ii) Find the x co-ordinate of the point of inflection of f.

$x =$ __________

(c) The diagram below shows the curve $y = p(x)$ and the line $y = l(x)$, for $0 \leq x \leq 10, x \in \mathbb{R}$.



There are two values of x in the domain $0 \leq x \leq 10$ for which:

$$p'(x) = l'(x)$$

where $p'(x)$ is the derivative of $p(x)$.

Use the infomation in the diagram to estimate these two values of x, as accurately as you can. Show your work on the diagram.

$x =$ __________ **or** ___________

Question 4 **(30 marks)**

(a) Differentiate the following function **from first principles**, with respect to x:

$$f(x) = x^2 - 7x - 10$$

(b) The function $g(x)$ is defined for $x \in \mathbb{R}$ by:

$$g(x) = \frac{6x+1}{x^4+3}$$

Find the value of $g'(-2)$, the derivative of $g(x)$ when $x = -2$.

(c) $h: \mathbb{R} \to \mathbb{R}$ is a continuous function.
The graph of $h(x)$ has a **local minimum** at the point $(0, 5)$.

State whether the following statement is true or false:

"The value of $h(x)$ must be **at least 5** for all real values of x."

Justify your answer.

Answer (true or false):

Justification:

Question 5 **(30 marks)**

(a) The first three terms of an arithmetic sequence are as follows, where $p \in \mathbb{R}$:

$$T_1 = 2p + 1$$
$$T_2 = 5p - 3$$
$$T_3 = 6p + 7$$

Find the value of p.

(b) $G_7 = 6$ and $G_{11} = \frac{3}{8}$ are the 7th and 11th terms of a geometric sequence, respectively.

Find the **two** possible values of r, the common ratio of this sequence, where $r \in \mathbb{R}$.

(c) A sequence of functions F_0 , F_1 , F_2 , ... is defined as follows, for $x \in \mathbb{R}$, $x > 0$:

- $F_0 = x^{2024}$
- For $n \geq 1$, the function F_n is the **derivative** of F_{n-1} with respect to x

(i) Write F_1 and F_2 in terms of x.

(ii) Find the first value of n for which $F_n = 0$.

Question 6 **(30 marks)**

(a) $h(x) = x^2 + bx - 12$, where $x \in \mathbb{R}$ and b is a constant.

Find the value of b for which $x - 4$ is a factor of $h(x)$.

(b) Two functions, $f(x)$ and $g(x)$, are defined as follows, for $x \in \mathbb{R}$, $x > 0$:

$$f(x) = e^{9x}$$
$$g(x) = \ln\sqrt{x}$$

Use these functions to answer parts **(b)(i)**, **(b)(ii)**, and **(b)(iii)**.

(i) Find the value of $f(1{\cdot}2)$.

Give your answer in the form $a \times 10^n$, where $a \in \mathbb{R}$, $1 \le a < 10$, $n \in \mathbb{N}$, and a is correct to 1 decimal place.

(ii) Find the value of x for which $g(x) = 3{\cdot}5$.
Give your answer in the form e^p, where $p \in \mathbb{R}$.

(iii) Write the function $g\big(f(x)\big)$ in terms of x, in its simplest form.

Section B	Contexts and Applications	150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

(a) Fiadh has a gross annual salary of €$54\ 000$.

She pays income tax at a rate of 20% on the first €$40\ 000$ of her salary and at a rate of 40% on the remainder. She has an annual tax credit of €1775.

Work out her net annual pay, assuming that there are no other deductions.

(b) Fiadh and her partner take out a 25-year mortgage with a monthly interest rate of $0{\cdot}279\%$.
They make equal monthly repayments of €$1647{\cdot}75$ at the end of each month.
They make the first repayment exactly one month after they take out the mortgage.

(i) Write down the present value of each of their first three monthly repayments, at the time when they take out the mortgage.

Give each value as a fraction. Do **not** multiply out any powers.

(ii) Work out the amount of money that Fiadh and her partner borrowed for their mortgage. Give your answer correct to the nearest euro.

(c) Fiadh puts money in a savings account, and leaves it there for a number of years. The following expression gives $F(t)$, the amount of money in the account in euro after t years, where $t \in \mathbb{R}$, $t \geq 0$:

$$F(t) = 5000\, e^{0 \cdot 04t}$$

Use this expression to answer parts **(c)(i)**, **(c)(ii)**, and **(c)(iii)**.

(i) Use differentiation to find the rate at which the amount of money in the account is increasing after $3 \cdot 5$ years. Give your answer correct to the nearest euro per year.

This question continues on the next page.

(ii) Use integration to find the average amount of money in the account over the first 5 years. Give your answer correct to the nearest euro.

Remember that the amount of money in the account after t years is:

$$F(t) = 5000\, e^{0 \cdot 04t}$$

(iii) Work out the annual rate of interest (AER) for this account.
That is, find the percentage increase in the amount of money in the account over the course of one year. Give your answer as a percentage, correct to 2 decimal places.

Question 8 **(50 marks)**

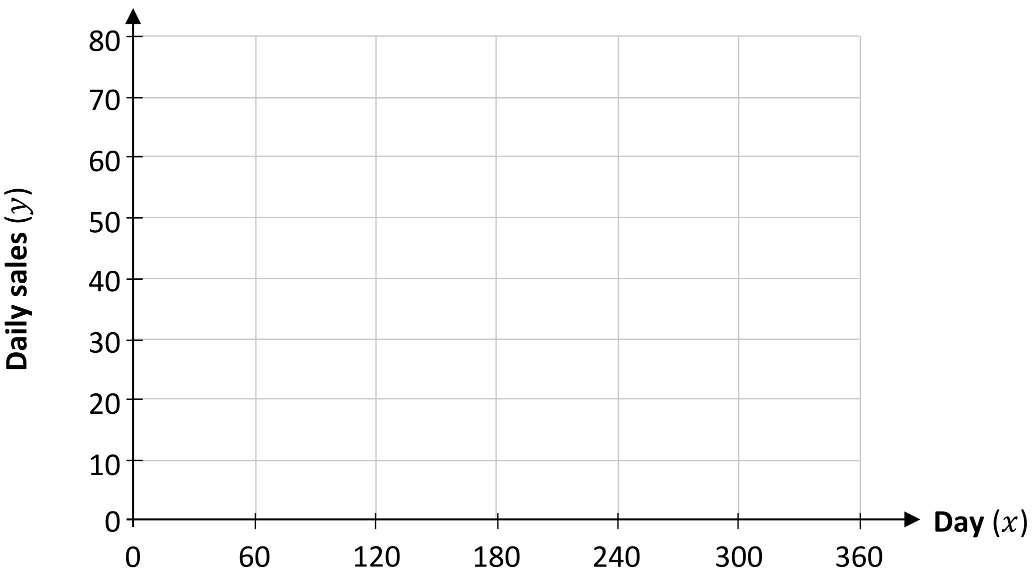
(a) A clothing company sells t-shirts. The daily sales of t-shirts over a 360-day period can be modelled using the following function, $T(x)$, where $0 \le x \le 360$, $x \in \mathbb{R}$:

$$T(x) = \left(\frac{x - 240}{60}\right)^3 + 70$$

(i) Fill in the table below, showing the value of $T(x)$ for each of the given values of x.
Give each value correct to the nearest whole number.
Two values have already been filled in.

x	0	60	120	180	240	300	360
$T(x)$		43					78

(ii) Draw the graph of $y = T(x)$ on the axes below, for $0 \le x \le 360$, $x \in \mathbb{R}$.

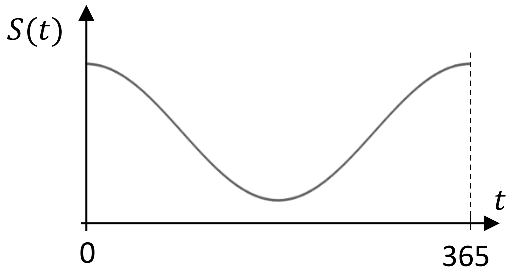


This question continues on the next page.

The clothing company also sells scarves. The daily sales of scarves can be modelled using the following function, $S(t)$, which is a periodic function with a period of 365 days:

$$S(t) = 21 + 19\cos\left(\frac{2\pi t}{365}\right)$$

Here, $t \in \mathbb{R}$ is the time, in days, from 1 January 2024, $t \geq 0$, and $\frac{2\pi t}{365}$ is in radians.



(b) Work out the maximum **and** minimum daily sales of scarves, according to $S(t)$.
You do **not** need to use differentiation.

Maximum = ________________ Minimum = ________________

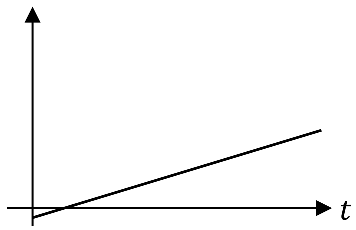
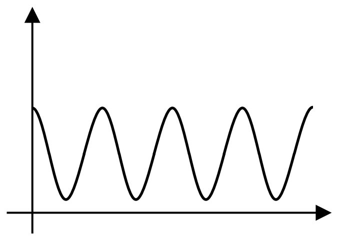
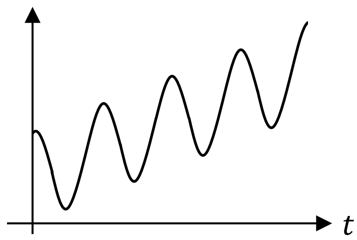
A different function, $C(t)$, can also be used to model the daily sales of scarves, where:

$$C(t) = S(t) - 2{\cdot}4 + 0{\cdot}03t$$

Again, $t \in \mathbb{R}$ is the time, in days, from 1 January 2024, $t \geq 0$, and $\frac{2\pi t}{365}$ is in radians.

(c) Find the value of t for which $S(t)$ and $C(t)$ give the same number of daily sales.

(d) The graphs, **J**, **K**, and **L**, of three functions are shown below (not to scale).

Graph J	**Graph K**	**Graph L**
		

One of these graphs shows the function $C(t)$ over a number of years, where $C(t) = S(t) - 2{\cdot}4 + 0{\cdot}03t$. Write down which graph this is. Justify your answer.

Answer (**J**, **K**, or **L**):

Justification:

(e) The derivative of $C(t)$ is given by $C'(t) = 0{\cdot}03 - \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right)$.

Find the value of t that gives the first local maximum of C, that is, the first time that $C'(t) = 0$. Give your answer correct to the nearest day.

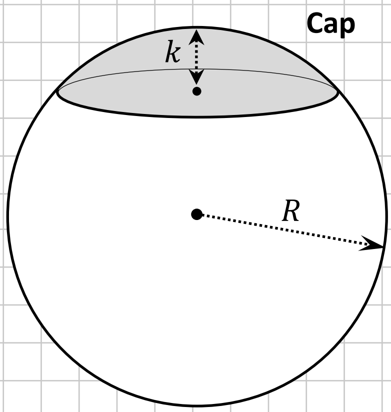
Question 9 **(50 marks)**

A sphere has a radius of R units.
The part of the sphere that is cut off by a flat surface is called a "cap", and has a volume of:

$$C = \frac{\pi\, k^2}{3}(3R - k)$$

Here, C is the volume of the cap and k is the height of the cap, with $0 < k \leq R$.

(a) Find the value of C, the volume of the cap, when $R = 13$ and $k = 4$.
Give your answer in terms of π.



(b) A different sphere has a radius of 8 units and a cap with a height of y units, with $0 < y \leq 8$.

The volume of this cap is $(36\pi\, y)$ units^2.

(i) Use this, and the expression for C above, to show that:

$$\frac{y}{3}(24 - y) = 36$$

(ii) Multiply out and solve the equation $\frac{y}{3}(24 - y) = 36$ to find the height of this cap.

Height of cap: ________________

A hemisphere has a diameter of x cm.
$V(x)$, the volume of this hemisphere in cm^3, is given by:

$$V(x) = \frac{\pi}{12}\, x^3$$

(c) Find the value of x when the volume of the hemisphere is 3 litres.
Give your answer correct to 1 decimal place.

This question continues on the next page.

(d) The volume of the hemisphere is increasing at a constant rate of 450 cm^3 per second.

Find the rate at which the diameter (x) of the hemisphere is increasing with respect to time, when $x = 20$ cm. Give your answer in cm per second, correct to 1 decimal place.

Remember that the volume of the hemisphere is $V(x) = \frac{\pi}{12}x^3$.

(e) A cone has a radius of r cm and a height of h cm.
The curved surface area of the cone, S, can be written as:

$$S = \pi r\sqrt{r^2 + h^2}$$

Rearrange this to write h in terms of S, r, and π.

Give your answer in the form $\dfrac{\sqrt{S^2 - ar^n}}{br}$, where a, b, and n are constants.

Question 10 **(50 marks)**

A company grows and sells plants.

(a) The function $W(x)$ is defined below. It can be used to model the height, in mm, of a water spinach plant, for the first 35 days after it starts to grow.

$$W(x) = 0{\cdot}667\,x + 1{\cdot}5\,x^2 - 0{\cdot}025\,x^3$$

Here, x is the number of days after the plant starts to grow, where $0 \leq x \leq 35$, $x \in \mathbb{R}$.

(i) Use $W(x)$ to estimate the height of a water spinach plant after 15 days.
Give your answer correct to the nearest mm.

(ii) Write down $W'(x)$, the derivative of $W(x)$.

(b) The height of a different plant can be modelled by the function $P(x)$, where x is again the number of days after the plant starts to grow.

The derivative of this function is:

$$P'(x) = 1{\cdot}1 + 2{\cdot}73\,x - 0{\cdot}078\,x^2$$

Find the range of values of x for which $P'(x) > 24$.
In your answer, give each value correct to the nearest whole number.

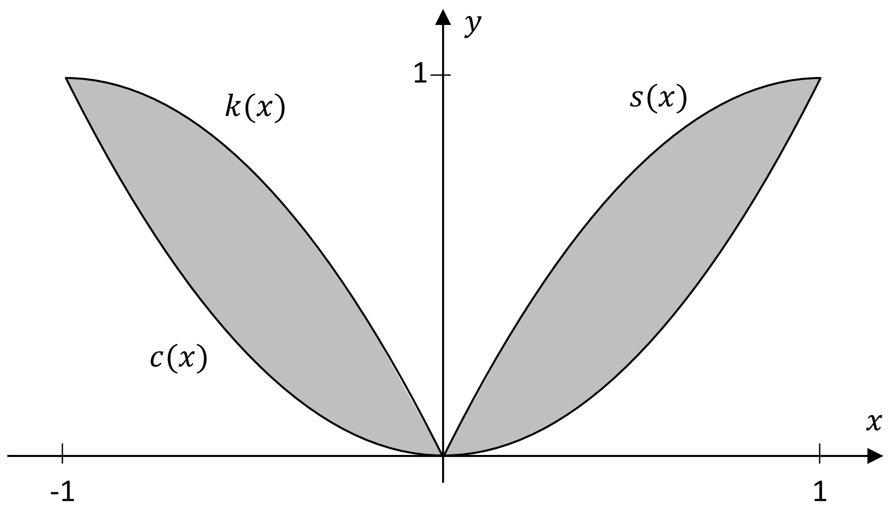
This question continues on the next page.

(c) The logo for the company is shown on the co-ordinate diagram below. The logo is the region enclosed by three curves, defined by the following functions c, s, and k :

$c(x) = x^2$, for $-1 \leq x \leq 1$, $x \in \mathbb{R}$

$s(x) = 2x - x^2$, for $0 \leq x \leq 1$, $x \in \mathbb{R}$

$k(x)$ is the image of $s(x)$ under axial symmetry in the y-axis.



(i) Use integration to work out the area of the logo.

Hint: find the area of the logo in the first quadrant, and then double it.

(ii) The function k can be written in the form $k(x) = -x^2 + bx + c$, where $b, c \in \mathbb{R}$ are constants. Find the value of b and the value of c.

Remember that k is the image of s under axial symmetry in the y-axis.

$b =$ ________________ $c =$ ________________

(d) In this part, p and r are constants, with $p, r \in \mathbb{R}$ and $0 < r < 0 \cdot 9\, p$.

The company sells bags of plant food.
The usual price of one bag is € p.

In a sale, the customer can choose to pay using either Option 1 or Option 2, as follows:

- **Option 1**: the usual price reduced by 10%, and then reduced by a further € r
- **Option 2**: the usual price reduced by € r, and then this new price reduced by 10%.

Which option (1 or 2), if either, is cheaper?
Write the price for each option (1 and 2) in terms of p and r, to support your answer.

Cheaper option:	Option 1	Option 2	It depends on the value of p and of r
(Tick (✓) **one** box only)	☐	☐	☐

Price for Option 1, in terms of p and r:

Price for Option 2, in terms of p and r:

Page for extra work.
Label any extra work clearly with the question number and part.

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