2013. M229

Coimisiún na Scrúduithe Stáit
State Examinations Commission

# Leaving Certificate Examination 2013

# Mathematics
# (Project Maths – Phase 2)

## Paper 1

## Higher Level

## Friday 7 June Afternoon 2:00 – 4:30

300 marks

| Examination number |
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| |

| Centre stamp |
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| Running total | |
|---|---|

| For examiner | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| | |
| Total | |

| Grade |
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## Instructions

There are **three** sections in this examination paper:

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|---|---|---|---|
| Section A | Concepts and Skills | 100 marks | 4 questions |
| Section B | Contexts and Applications | 100 marks | 2 questions |
| Section C | Functions and Calculus (old syllabus) | 100 marks | 2 questions |

Answer all eight questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

| Section A | Concepts and Skills | 100 marks |
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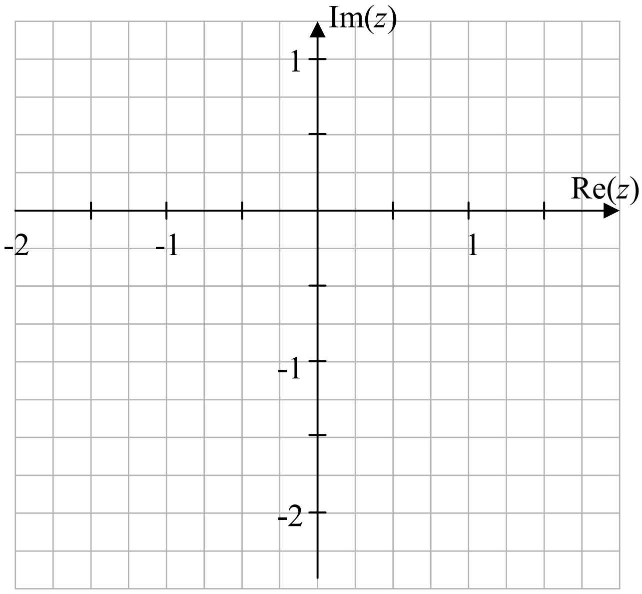
Answer **all four** questions from this section.

**Question 1** **(25 marks)**

$z = \dfrac{4}{1+\sqrt{3}i}$ is a complex number, where $i^2 = -1$.

**(a)** Verify that $z$ can be written as $1-\sqrt{3}i$.

**(b)** Plot $z$ on an Argand diagram and write $z$ in polar form.




**(c)** Use De Moivre's theorem to show that $z^{10} = -2^9\left(1-\sqrt{3}i\right)$.

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**Question 2** **(25 marks)**

**(a)** Find the set of all real values of $x$ for which $2x^2 + x - 15 \geq 0$.

**(b)** Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

**Question 3** **(25 marks)**

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon−14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0 \cdot 693t}{5730}}$, where $Q$ is the proportion of carbon−14 remaining and $t$ is the age, in years, of the item.

**(a)** An item is 2000 years old. Use the formula to find the proportion of carbon−14 in the item.

**(b)** The proportion of carbon−14 in an item found at Lough Boora, County Offaly, was 0·3402. Estimate, correct to two significant figures, the age of the item.

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**Question 4** **(25 marks)**

**(a)** Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.

**(i)** Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0·327%, correct to 3 decimal places.

**(ii)** Niamh has €15 000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

**(b)** Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0·866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

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| Section B | Contexts and Applications | 100 marks |
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Answer **both** question 5 **and** question 6 from this section.

**Question 5** **(50 marks)**

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

**(a)** If the ticket price was €18, how many people would be expected to attend?

**(b)** Let $x$ be the ticket price, where $x \leq 20$. Write down, in terms of $x$, the expected attendance at such an event.

**(c)** Write down a function $f$ that gives the expected income from the sale of tickets for such an event.

**(d)** Find the price at which tickets should be sold to give the maximum expected income.

**(e)** Find this maximum expected income.

**(f)** Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

**(g)** The stadium was full for a recent special event. Two types of tickets were sold, a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold, the income from the event would have been reduced by €14 000. How many family tickets were sold?

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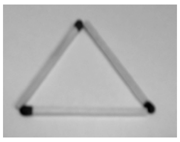
**Question 6** **(50 marks)**

Shapes in the form of small equilateral triangles can be made using matchsticks of equal length. These shapes can be put together into patterns. The beginning of a sequence of these patterns is shown below.

1



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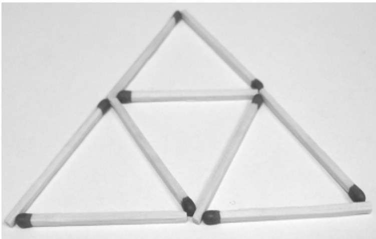

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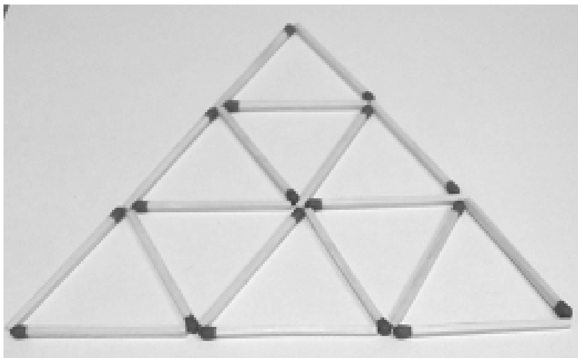

**(a)** **(i)** Draw the fourth pattern in the sequence.

**(ii)** The table below shows the number of small triangles in each pattern and the number of matchsticks needed to create each pattern. Complete the table.

| Pattern | $1^{st}$ | $2^{nd}$ | $3^{rd}$ | $4^{th}$ |
|---|---|---|---|---|
| Number of small triangles | 1 | | 9 | |
| Number of matchsticks | 3 | 9 | | |

**(b)** Write an expression in $n$ for the number of triangles in the $n^{th}$ pattern in the sequence.

**(c)** Find an expression, in $n$, for the number of matchsticks needed to turn the $(n-1)^{\text{th}}$ pattern into the $n^{\text{th}}$ pattern.

**(d)** The number of matchsticks in the $n^{\text{th}}$ pattern in the sequence can be represented by the function $u_n = an^2 + bn$ where $a,\ b \in \mathbb{Q}$ and $n \in \mathbb{N}$. Find the value of $a$ and the value of $b$.

**(e)** One of the patterns in the sequence has 4134 matchsticks. How many small triangles are in that pattern?

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## Section C — Functions and Calculus (old syllabus) — 100 marks

Answer **both** Question 7 **and** Question 8 from this section.

**Question 7** **(50 marks)**

**(a)** Differentiate $\dfrac{5x}{x+4}$, with respect to $x$ for $x \neq -4$.

**(b)** A curve is defined by the parametric equations

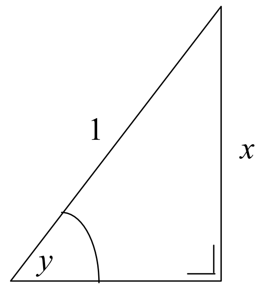
$$x = 1 + e^{-t}, \qquad y = t^2 + 2e^t.$$

**(i)** Show that $\dfrac{dy}{dx} = -2e^t\left(t + e^t\right)$.

**(ii)** Hence, find the equation of the tangent to the curve at the point $x = 2$.

**(c)** **(i)** Write $x$ in terms of $\sin y$, using the diagram. Hence, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$




**(ii)** If $y = x + \sin^{-1} x$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + x = 0.$

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**Question 8** **(50 marks)**

**(a)** Evaluate $\int_0^2 12e^{3x}\,dx$ and give your answer in the form $a(e^b - 1)$.

**(b)** The function $f(x) = x^3 + ax^2 + bx$ has turning points at $x = 2$ and $x = -\frac{4}{3}$.

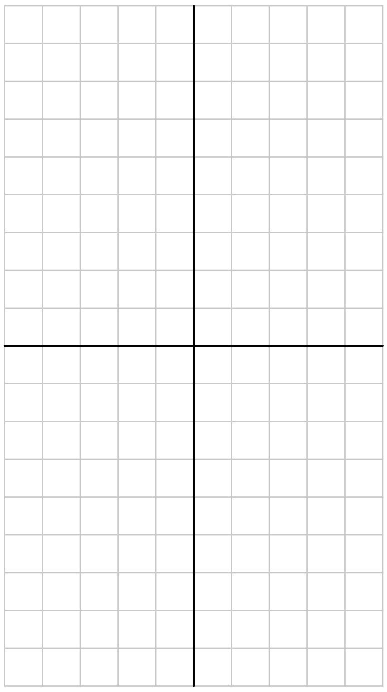
**(i)** Find the value of *a* and the value of *b*.

**(ii)** Find the co-ordinates of the turning points and hence draw a sketch of the curve $y = f(x)$.

**(c)** **(i)** Draw the graphs of

$$y = 4x \qquad \text{and} \qquad y = x^3$$

in the domain $-2 \leq x \leq 2, \ \ x \in \mathbb{R}$.



**(ii)** Find the area of the region in the first quadrant enclosed by the two graphs.

**(iii)** Write down the total area enclosed between the two graphs and give a reason for your answer.

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