Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2019

Mathematics

Paper 1
Higher Level

Friday 7 June
Afternoon 2:00 to 4:30

300 marks

Examination Number				

Centre Stamp

Running Total	

For Examiner					
Q.	Ex.	Adv. Ex.	Q.	Ex.	Adv. Ex.
1			11		
2			12		
3			13		
4			14		
5					
6					
7					
8					
9					
10			Total		

Grade

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 **(Suggested maximum time: 5 minutes)**

There are 85 students in third year in Liam's school.

(a) $\frac{2}{5}$ of these students do Business Studies.

Work out the number of these students who do **not** do Business Studies.

(b) 26 of the students in third year do Art.

Work out the **percentage** of students in third year who do Art, correct to one decimal place.

The ratio of students to teachers in Liam's school was $15{:}1$.
The school hired one extra teacher, while the number of students stayed the same.
The new ratio of students to teachers was $a{:}1$, where $a \in \mathbb{Q}$.

(c) Put a tick (✓) in the correct box to show which statement is true. Tick **one** box only.
Give an example to support your answer.

$a > 15$	$a = 15$	$a < 15$
☐	☐	☐

Example:

Question 2 **(Suggested maximum time: 10 minutes)**

(a) Write down the four factors of 45, apart from 1 and 45.

Answer: ☐, ☐, ☐, ☐

(b) n is a whole number bigger than 1. It has just two factors: 1 and n.
Put a tick (✓) in the correct box to show what name is given to this type of number.
Tick **one** box only.

composite	prime	square
☐	☐	☐

(c) p is a whole number. Apart from 1 and p, the only other factor that p has is 7.
Work out the value of p.

(d) Write down the four factors of $12k + 8$, apart from 1 and $12k + 8$.
Two of the factors should be in terms of k.

Answer: ☐, ☐, ☐, ☐

Question 3 **(Suggested maximum time: 5 minutes)**

(a) The distance between the Earth and Mars is at least $56\,000\,000$ km.
Write this distance in the form $a \times 10^n$ km, where $1 \leq a < 10$ and $n \in \mathbb{N}$.

(b) The diameter of a human hair is roughly $0{\cdot}0075$ cm.
Write this diameter in the form $b \times 10^n$ cm, where $1 \leq b < 10$ and $n \in \mathbb{Z}$.

(c) Lewis was driving at 90 km per hour when he sneezed.
During the sneeze, his eyes were closed for half a second.
Work out how many metres he travelled in this time.

Question 4 **(Suggested maximum time: 10 minutes)**

(a) Katie has a gross annual income of $€52\,460$.
$8{\cdot}5\%$ of this is **deducted** in pension contributions.
The amount that is left is Katie's **taxable** income.

(i) Work out Katie's taxable income, after the pension contributions have been deducted.

Katie pays income tax on her **taxable** income at a rate of 20% on the first $€34\,000$, and 40% on the balance. She has annual tax credits of $€4200$.

(ii) Work out Katie's net income, after income tax has been deducted.

(b) Katie got a credit card bill 3 months ago for $€420$.
The interest rate on her credit card is 2% per month, compounded monthly.
She has not paid off any of her bill.

Work out what her bill is now. Give your answer in euro, correct to the nearest cent.

(c) Katie bought a motorbike last year.
Since she bought it, the motorbike lost 10% of its value.
The value of the motorbike is now $€12\ 150$.

Work out the value of the motorbike when Katie bought it.

Question 5 **(Suggested maximum time: 10 minutes)**

The sets A, B, and C are as follows:

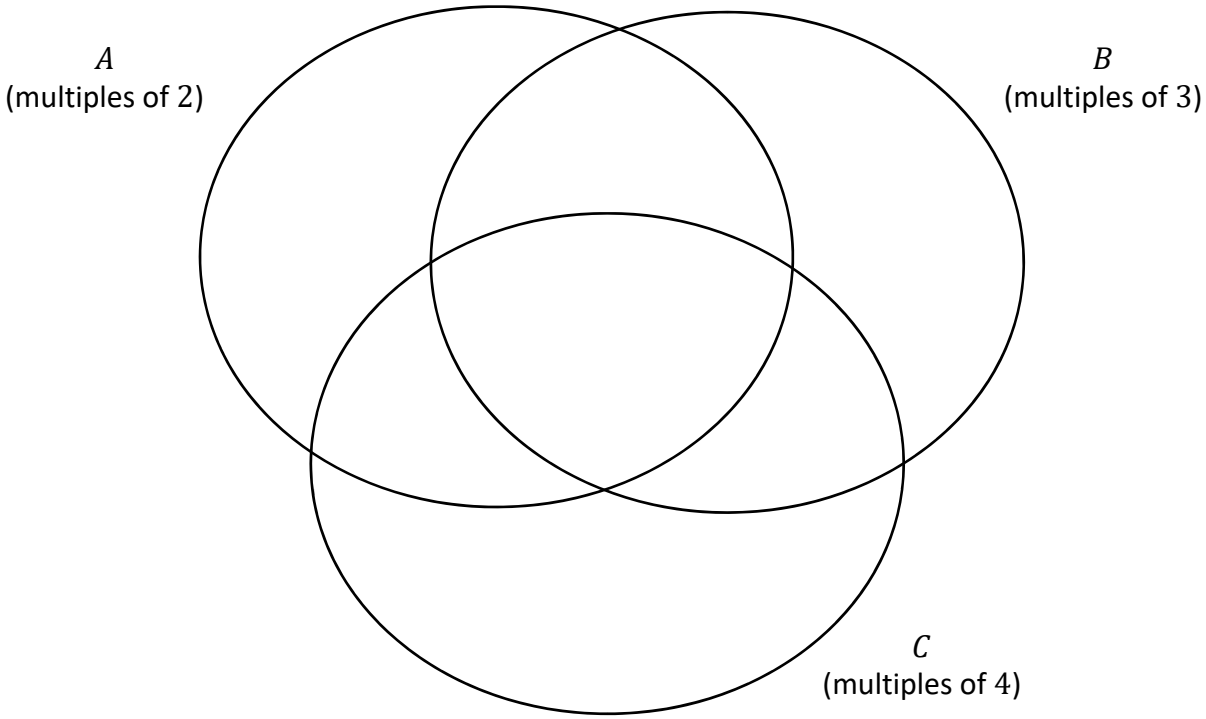
A is the set of multiples of $2 = \{2, 4, \dots\}$
B is the set of multiples of $3 = \{3, 6, \dots\}$
C is the set of multiples of $4 = \{4, 8, \dots\}$.

(a) Write down a number that is in $A \cap B \cap C$.

Answer:

(b) Explain why C is a subset of A.

These sets A, B, and C are shown in the Venn diagram below.
There are 7 different regions in the Venn diagram.



(c) Because C is a subset of A, there are two regions in the Venn diagram that have **no** elements. Write an **X** in each of these two regions in the Venn diagram above.

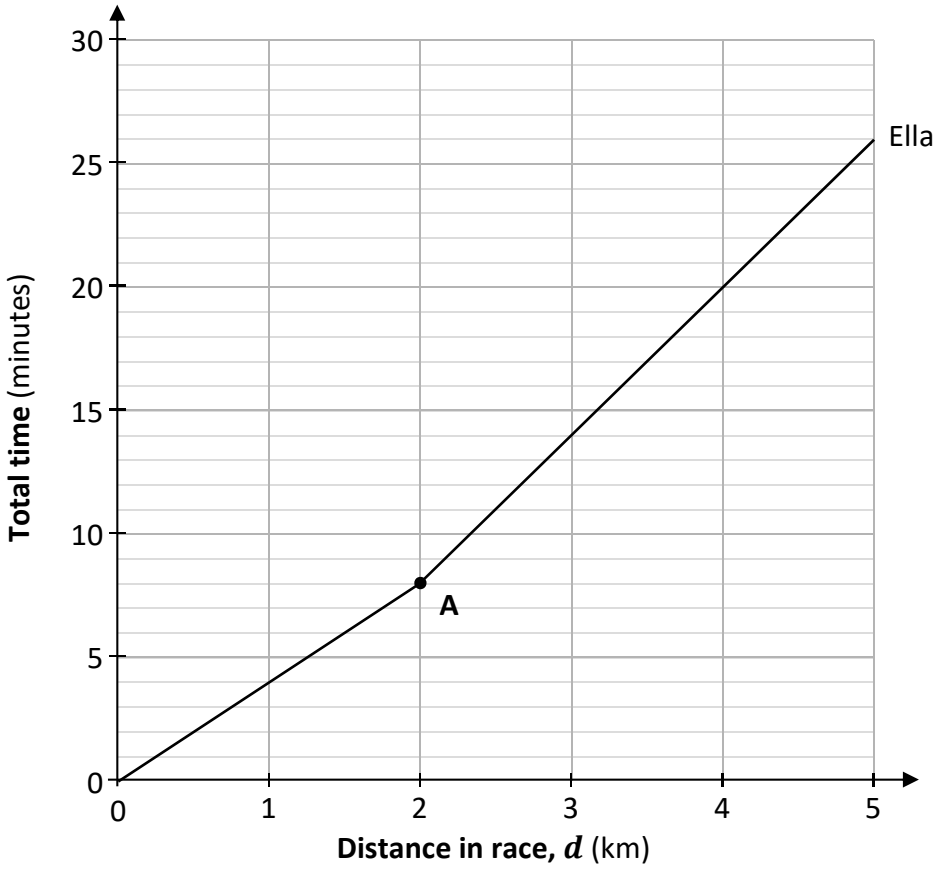
(d) Each of the other five regions in the Venn diagram has some elements.

In **each** of these five regions **in the Venn diagram above**, write one of the elements in that region.

Question 6 **(Suggested maximum time: 20 minutes)**

Poppy and Ella ran a 5 km race. The simplified graph below shows the time that it took Ella to run d km during the race. One of the points on the graph is marked **A**.

Distance is on the horizontal axis so, for example, it took Ella 26 minutes to run the whole 5 km.



The table below shows the total time that it took Poppy and Ella to run each of the given distances in the race.

(a) Using the figures in the table, **draw** a graph on the diagram above to show the time it took **Poppy** to run d km during the race, for $0 \leq d \leq 5$ and $d \in \mathbb{R}$.

(b) Using **Ella**'s graph, fill in the three missing values in the table below.

Distance in the race (km)	**Total time taken for Poppy** (minutes)	**Total time taken for Ella** (minutes)
1	5	4
2	10	
3	17	
4	24	
5	30	26

(c) Show that **Poppy**'s times in the table do **not** make a quadratic sequence.

(d) It took Ella 26 minutes to run the 5 km. Work out Ella's **average speed** for the race. Give your answer in km per hour, correct to two decimal places.

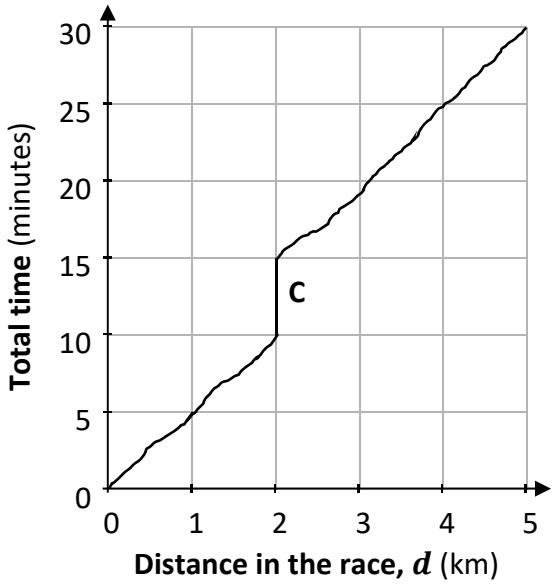
(e) Tick (✓) the correct box to show what happened **Ella**'s speed after 2 km, which is marked **A** on the graph. Tick **one** box only. Justify your answer.

Ella's speed increased	Ella's speed decreased	Ella's speed stayed the same
☐	☐	☐

Justification:

This question continues on the next page.

Ciarán also ran the 5 km race. He drew the graph below to show the time that it took him to run d km during the race. The part of Ciarán's graph marked **C** is a vertical line.



(f) What does the part **C** tell us about Ciarán's running at this stage of the race?
Give as much detail as possible.

(g) Brendan says: "Ciarán's graph does **not** show total time as a **function** of distance (d)".
Give a reason why Brendan is correct.

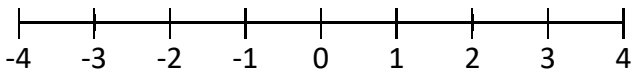
Question 7 **(Suggested maximum time: 5 minutes)**

(a) Describe each of the following sets. Be as specific as possible.

(i) The set of natural numbers, $\mathbb{N}$.

(ii) The set of integers, $\mathbb{Z}$.

(b) Graph the following inequality on the number line given.

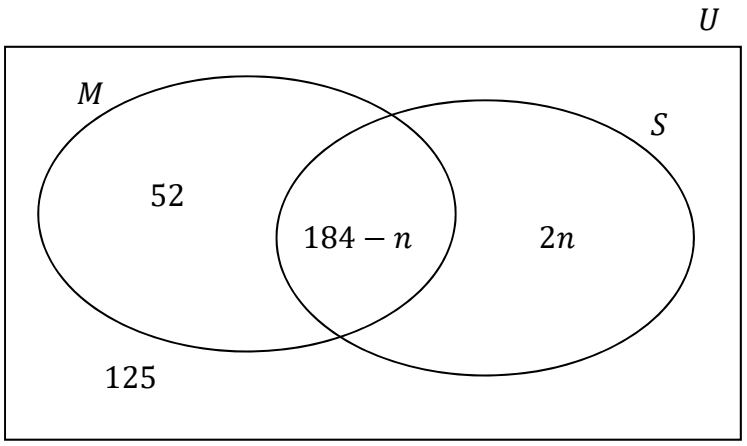
Inequality	Number line
$-3 < x \leq 2$, where $x \in \mathbb{R}$	

(c) Use algebra to solve the following inequality:

$$-7 < 8 - 3g \leq 11$$

Question 8 **(Suggested maximum time: 5 minutes)**

The Venn diagram below shows the number of people in a youth club (U) who play music (M) and sport (S), where $n \in \mathbb{N}$.



Work out the **maximum** number of people who could be in the youth club.

Answer:

Question 9 **(Suggested maximum time: 20 minutes)**

Gertie writes down the following sequence, which repeats every three terms:

$$3,\ 6,\ 4,\ 3,\ 6,\ 4,\ 3,\ \ldots$$

The 1st term is 3.

(a) **(i)** Write down the value of the 12th term.

Answer:

(ii) Work out the value of the 100th term in this sequence.

Answer:

(b) Describe how to find the value of the nth term in the sequence, where $n \in \mathbb{N}$, **without** listing all the terms from the 1st to the nth.

This question continues on the next page.

Gertie made her sequence $3, 6, 4, 3, 6, 4, 3, \ldots$ by picking 3 as the 1st term, and then using **this rule**:

> If a term is **odd**, multiply it by 2 to get the next term.
>
> If a term is **even**, add 2 to it and half your answer to get the next term.

For example, 3 is odd, so the next term is 2×3, which is 6.
6 is even, so the next term is $\frac{1}{2} \times (6 + 2)$, which is 4.

(c) A different sequence follows the **same rule**, but has **8 as the 1st term**.
Work out the next four terms of this sequence.

Answer: 8 , ____ , ____ , ____ , ____

Working out:

$$\text{2nd term} = \frac{1}{2} \times (8 + 2) =$$

(d) Ahmed takes **2 as his 1st term**, and makes a sequence using the **same rule**.

State what is unusual about Ahmed's sequence.
It might be helpful to work out some of the terms of his sequence.

(e) In another sequence using the **same rule**, the **2nd term is 86**.
Work out the **two** different values that the 1st term could have in this sequence.

Answer: [ ], [86], … **or** [ ], [86], …

Working out:

(f) A different sequence following the **same rule** starts with the number k, which is **odd**.

Work out the next three terms of this sequence.
Give each term in its simplest form in terms of k.

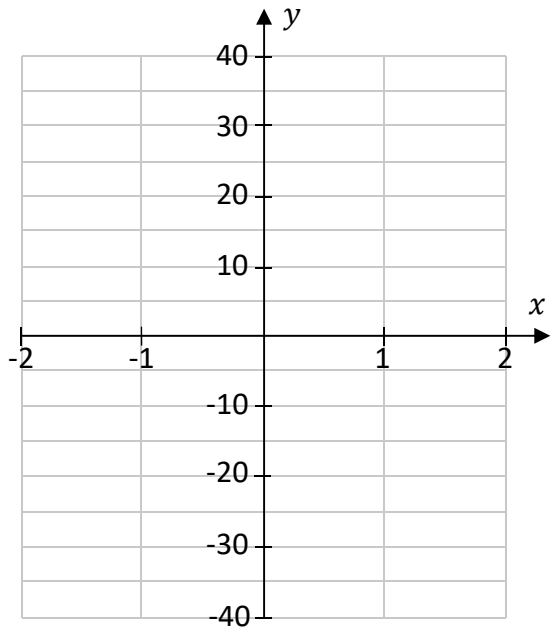
Answer: [k], [ ], [ ], [ ]

Working out:

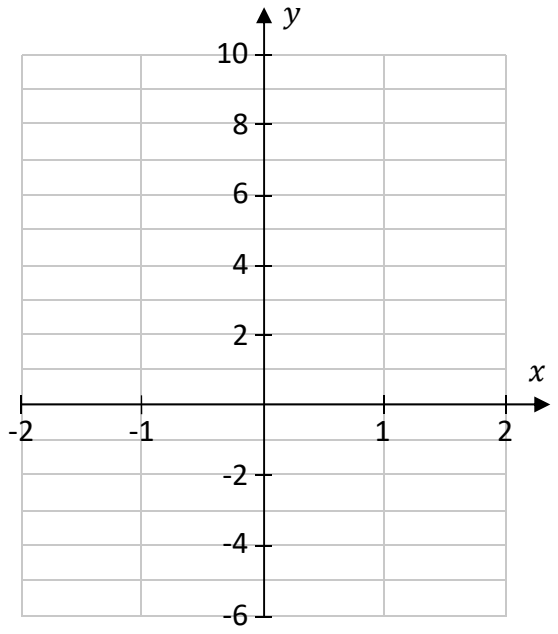
Question 10 **(Suggested maximum time: 10 minutes)**

Draw each of the following two functions in the domain $-2 \leq x \leq 2$, for $x \in \mathbb{R}$.
Show your working out.

Function: $y = 10x - 4x^2$



Function: $y = 3^x$



Question 11 **(Suggested maximum time: 5 minutes)**

Solve the following equation. Give your answer in the form $\frac{m}{n}$ where $m, n \in \mathbb{N}$.

$$\frac{3x+5}{2}+\frac{x-4}{3}=16$$

Question 12 **(Suggested maximum time: 10 minutes)**

(a) Factorise $a^2 - 16n^2$.

(b) One of the factors of $8x^2 + 45x - 18$ is $x + 6$.

(i) Factorise $8x^2 + 45x - 18$.

(ii) Write down one quadratic expression in x, other than $8x^2 + 45x - 18$, that has $x + 6$ as a factor.

Give your answer in the form $ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

Answer:

(c) Show that $2y + 3$ is a factor of $2y^3 - 9y^2 - 28y - 15$.

Question 13 **(Suggested maximum time: 15 minutes)**

Freda is starting an exercise program. She wants to increase her power output (P).
One formula for P is:

$$P = 36w + 62h + 1800$$

where w is Freda's weight (in kilograms) and h is the height (in metres) that she can jump.

(a) Work out the value of P when $w = 70$ kg and $h = 0{\cdot}65$ m.

After one week, Freda's power output (P) has **increased by 8**. Her weight (w) has not changed.

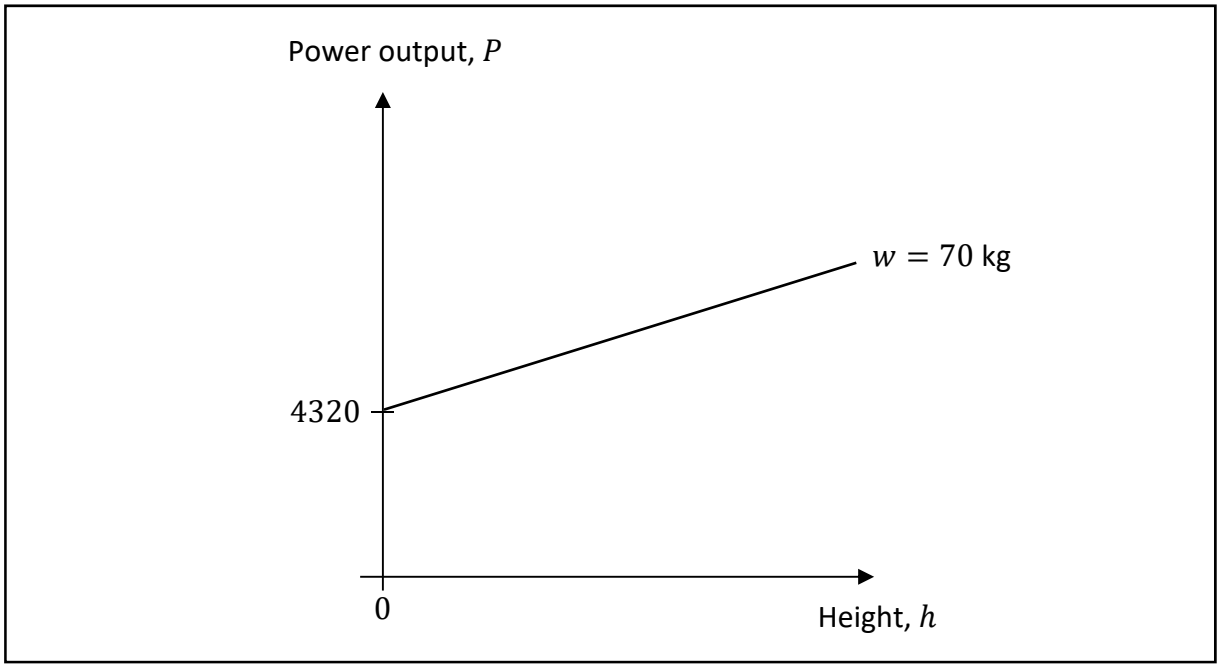
(b) Work out the new value of h after this week, correct to two decimal places.

The graph below shows the value of P for different heights (h) when $w = 70$ kg.
This graph cuts the y-axis when $P = 4320$, as shown.

(c) **Draw** a graph to show the values of P for different heights (h) when $\boldsymbol{w = 80}$ **kg**, using the same axes, scales, and domain.

State on the diagram the value of P where your graph cuts the y-axis.

Note: the axis with P does not start at 0.



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Freda finds a second formula for power output. She sets the two formulas equal.
This gives the following identity, where f is Freda's height (in metres):

$$60w + 80h - 15f - 1300 = 36w + 62h + 1800$$

(d) Use this to write h in terms of w and f.

Answer: $h =$

Question 14 **(Suggested maximum time: 10 minutes)**

The first three terms of a sequence are:

Term 1:	Term 2:	Term 3:
$6x - 3$	$x^2 - 2x$	$4x^2 + 3x$

(a) Fill in the following table to write each of the **first differences** in its simplest form in terms of x.

First Difference 1: Term 2 – Term 1	**First Difference 2: Term 3 – Term 2**
$x^2 - 2x - (6x - 3) =$	

(b) Show that, if the terms form a linear sequence, then $2x^2 + 13x - 3 = 0$.

This question continues on the next page.

(c) Solve the equation $2x^2 + 13x - 3 = 0$. Give each answer correct to three decimal places.

Answer: [ ] and [ ]

Page for extra work.
Label any extra work clearly with the question number and part.

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